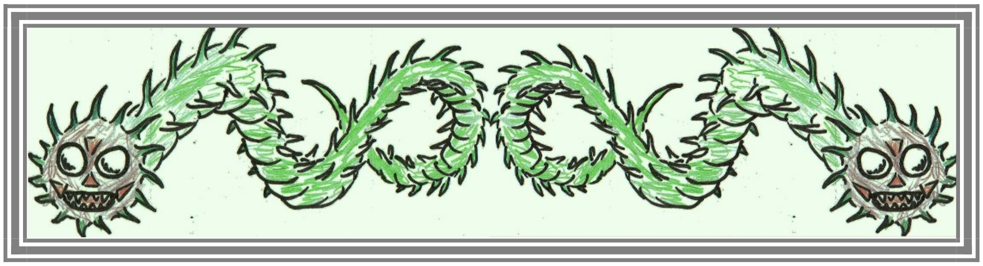

Translation of Maxwell's Equations Maxwell's Treatise Volume I, Preliminary, Article 17

Hamilton's Expression for the Relation Between a Force and It's Potential

Rev 0.00
Jim Satterwhite

James Satterwhite PE – NC Ret.

4/10/2025

Table of Contents

Table of Figures

Preliminary

Article 17

1. Hamilton's Expression for the Relation Between a Force and It's Potential

17.] The geometrical nature of the
tial and the vector thus derived from

The geometrical nature of the relation between the potential and the vector thus derived from it receives great light from Hamilton's discovery of the form of the operator by which the vector is derived from the potential.

The resolved part of the vector in
seen, the first derivative of the pote

The resolved part of the vector in any direction is, as we have seen, the first derivative of the potential with respect to a coordinate drawn in that direction, the sign being reversed.

Now if i, j, k are three unit vect
other, and if X, Y, Z are the componen
parallel to these vectors, then

$$\mathfrak{F} = iX + jY + kZ$$

Now if $i,\ j,\ k$ $\left(\hat{x},\ \hat{y},\ \hat{z}\right)$ are three unit vectors at right angles to each other, and if X, Y, Z $\left(F_x, F_y, F_z\right)$ are components of the vector $\mathfrak{F}$ $\left(\vec{F}\right)$ resolved parallel to these vectors, then

$$\mathfrak{F} = \mathrm{i}X + \mathrm{j}Y + \mathrm{k}Z; \qquad (1)$$

$$\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}; \qquad (1s)$$

and by what we have said above, if ψ $\left(U\right)$ is the potential

$$\mathfrak{F} = -\left(\mathrm{i}\frac{d\psi}{dx} + \mathrm{j}\frac{d\psi}{dy} + \mathrm{k}\frac{d\psi}{dz} \right); \qquad (2)$$

$$\vec{\mathrm{F}} = -\left(\frac{dU}{dx}\hat{x} + \frac{dU}{dy}\hat{y} + \frac{dU}{dz}\hat{z} \right); \qquad (2s)$$

If we now write ∇ for the operator,

$i\frac{d}{d} + j\frac{d}{d} + k$

If we now write ∇ $\left(\vec{\nabla}\right)$ for the operator,

$$\mathrm{i}\frac{d}{dx} + \mathrm{j}\frac{d}{dy} + \mathrm{k}\frac{d}{dz}, \qquad (3)$$

$$\frac{d}{dx}\hat{x} + \frac{d}{dy}\hat{y} + \frac{d}{dz}\hat{z}, \qquad (3s)$$

$$\mathfrak{F} = -\nabla\psi, \qquad (4)$$

$$\vec{\mathrm{F}} = -\vec{\nabla}U, \qquad (4s)$$

The symbol of operation ∇ may be
to measure, in each of three rectang
increase of Ψ, and then, considering t
vectors, to compound them into one.
to do by the expression (3). But we ma

The symbol of operation ∇ $\left(\vec{\nabla}\right)$ may be interpreted as directing us to measure, in each of three rectangular direction, the rate of increase of ψ $\left(U\right)$, and then, considering the quantities thus found as vectors, to compound them into one. This is what we are directed to do by expression (3). But we may also consider it as directing us first to find out in what direction ψ $\left(U\right)$ increases fastest, and then to lay off in that direction a vector representing this rate of increase.

M. Lamé, in his *Traité des Foncti*
Differential Parameter to express the
rate of increase, but neither the term it
Lamé uses it, indicates that the quan
to indicate the direction, as well as

M. Lamè, in his *Traitè des Fonctions* Inverses, uses the term Differential Parameter to express the magnitude of this greatest rate of increase, but neither the term itself, nor the mode in which Lamè uses it indicates that the quantity referred to has direction as well as magnitude. On those rare occasions in which I shall call the vector $\mathfrak{F}$ $\left(\vec{F}\right)$ the Slope of the scalar functions ψ $\left(U\right)$ using the word Slope to indicate it's direction, as well as the magnitude, of the most rapid decrease of ψ $\left(U\right)$.

2. Hamilton's Expression between for the Relation between a Force and its Potential

The geometrical nature of the relation between the potential and the vector thus derived from it receives great light from Hamilton's discovery of the form of the operator by which the is derived from the potential.

17.] The geometrical nature of the
tial and the vector thus derived from

The resolved part of the vector in any direction is , as we have seen, the first derivative of the potential with respect to a coordinate drawn in that direction , the sign being reversed.

The resolved part of the vector in a
seen, the first derivative of the pote

Now if $\hat{x}, \hat{y}, \hat{z}$ are three unit vectors at right angles to each other, and if F_x, F_y, F_z are the components of the vector $\vec{F}$ resolved parallel to these vectors, then

$$\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}; \qquad (1)$$

and by what we have said above, if ϕ is the potential,

$$\vec{F} = -\left(\frac{d\phi}{dx}\hat{x} + \frac{d\phi}{dy}\hat{y} + \frac{d\phi}{dz}\hat{z}\right) \qquad (2)$$

Now if i, j, k are three unit vect
other, and if X, Y, Z are the componen
parallel to these vectors, then

$$\mathfrak{F} = iX + jY + k$$

If we now write $\vec{\nabla}$ for the operator,

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \qquad (3)$$

$$\vec{F} = -\vec{\nabla}\phi \qquad (4)$$

If we now write ∇ for the operator,

$$i\frac{d}{\ } + j\frac{d}{\ } + k$$

The symbol of operation $\vec{\nabla}$ may be interpreted as directing us to measure, in each of the three rectangular directions, the rate of increase of ϕ, and then, considering the quantities thus found as vectors, to compound them into one. This is what we are directed to do by the expression (3). But we may also consider it as directing is first to find out in what direction ϕ increases fastest, and then to lay off in that direction a vector representing this rate of increase.

The symbol of operation ∇ may be
to measure, in each of three rectang
increase of Ψ, and then, considering t
vectors, to compound them into one.
to do by the expression (3). But we m

M. Lamé, in his *Traité des Fontions Inverses*, uses the term Differential Parameter to express the magnitude of this greatest rate of increase, but neither the term itself, nor the mode in which Lamé uses it, indicates that the quantity referred to has direction as well as magnitude. On those rare occasions in which I shall have to refer to this relation as a purely geometrical one, I shall call the vector $\vec{F}$ the Slope of the scalar function ϕ, using the word Slope to indicate the direction, as well as the magnitude, of the most rapid increase of ϕ.

M. Lamé, in his *Traité des Foncti*
Differential Parameter to express the
rate of increase, but neither the term it
Lamé uses it, indicates that the quant
to indicate the direction, as well as

Blackadder ITC ABCDEFGHIJKLMNOPQRSTWXYZ

Brush Script MT ABCDEFGHIIJKLMNOPQRSTUVWXY[

Euclid Fractur ABCDEFGHIJKLMNOPQRSTUVWXYZ

End of Document

D:\!\Maxwell\Translation\Art17\TELIMaxwellTranlationArt17-R_1.doc