



Translation of Maxwell's Equations Maxwell's Treatise Volume I, Preliminary, Article 17

*Hamilton's Expression for the Relation Between a
Force and It's Potential*

Rev 0.02
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Preliminary

Article 17

17. Hamilton's Expression for the Relation Between a Force and It's Potential

17.] The geometrical nature of the relation between the potential and the vector thus derived from it receives great light from Hamilton's discovery of the form of the operator by which the vector is derived from the potential.

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The resolved part of the vector in any direction is, as we have seen, the first derivative of the potential with respect to a co-ordinate drawn in that direction, the sign being reversed.

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Now if i, j, k are three unit vectors at right angles to each other, and if X, Y, Z are the components of the vector \mathfrak{F} resolved parallel to these vectors, then

$$\mathfrak{F} = iX + jY + kZ; \quad (1)$$

and by what we have said above, if Ψ is the potential,

$$\mathfrak{F} = -\left(i\frac{d\Psi}{dx} + j\frac{d\Psi}{dy} + k\frac{d\Psi}{dz}\right). \quad (2)$$

Now if i, j, k ($\hat{x}, \hat{y}, \hat{z}$) are three unit vectors at right angles to each other, and if X, Y, Z (F_x, F_y, F_z) are components of the vector \mathfrak{F} (\vec{F}) resolved parallel to these vectors, then

$$\mathfrak{F} = iX + jY + kZ; \quad (1)$$

$$\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}; \quad (1s)$$

and by what we have said above, if ψ (U) is the potential



$$\mathfrak{F} = -\left(i \frac{d\psi}{dx} + j \frac{d\psi}{dy} + k \frac{d\psi}{dz} \right); \quad (2)$$

$$\vec{F} = -\left(\frac{dU}{dx} \hat{x} + \frac{dU}{dy} \hat{y} + \frac{dU}{dz} \hat{z} \right); \quad (2s)$$

If we now write ∇ for the operator,

$$i \frac{d}{dx} + j \frac{d}{dy} + k \frac{d}{dz}, \quad (3)$$

$$\mathfrak{F} = -\nabla \Psi. \quad (4)$$

If we now write ∇ ($\vec{\nabla}$) for the operator,

$$i \frac{d}{dx} + j \frac{d}{dy} + k \frac{d}{dz}, \quad (3)$$

$$\frac{d}{dx} \hat{x} + \frac{d}{dy} \hat{y} + \frac{d}{dz} \hat{z}, \quad (3s)$$

$$\mathfrak{F} = -\nabla \psi, \quad (4)$$

$$\vec{F} = -\vec{\nabla} U, \quad (4s)$$

The symbol of operation ∇ may be interpreted as directing us to measure, in each of three rectangular directions, the rate of increase of Ψ , and then, considering the quantities thus found as vectors, to compound them into one. This is what we are directed to do by the expression (3). But we may also consider it as directing us first to find out in what direction Ψ increases fastest, and then to lay off in that direction a vector representing this rate of increase.

The symbol of operation ∇ ($\vec{\nabla}$) may be interpreted as directing us to measure, in each of three rectangular direction, the rate of increase of ψ (U), and then, considering the quantities thus found as vectors, to compound them into one. This is what we are directed to do by expression (3). But we may also consider it as directing us first to find out in what direction ψ (U) increases fastest, and then to lay off in that direction a vector representing this rate of increase.



M. Lamé, in his *Traité des Fonctions Inverses*, uses the term Differential Parameter to express the magnitude of this greatest rate of increase, but neither the term itself, nor the mode in which Lamé uses it, indicates that the quantity referred to has direction as well as magnitude. On those rare occasions in which I shall have to refer to this relation as a purely geometrical one, I shall call the vector \mathfrak{F} the Slope of the scalar function Ψ , using the word Slope to indicate the direction, as well as the magnitude, of the most rapid decrease of Ψ .

M. Lamé, in his *Traité des Fonctions Inverses*, uses the term Differential Parameter to express the magnitude of this greatest rate of increase, but neither the term itself, nor the mode in which Lamé uses it indicates that the quantity referred to has direction as well as magnitude. On those rare occasions in which I shall call the vector $\mathfrak{F}(\vec{F})$ the Slope of the scalar functions $\psi(U)$ using the word Slope to indicate it's direction, as well as the magnitude, of the most rapid decrease of $\psi(U)$.

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